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ACTUARIAL METHODS FOR CALCULATING INSURANCE TARIFFS IN THE CONTEXT OF FUTURE LOSS RESERVES

Summary

Relevance. Problem statement. In up-to-date reality, the shortcomings of existing methods for estimating insurance loss reserves are highly probable, especially regarding the further application of multifactor models for tariff calculation. Therefore, it is necessary to propose an approach to reserving losses for actuarial pricing based on generalized linear models. Furthermore, it is crucial to address the problem of loss reserves distribution, estimated by actuaries using well-known reserving models, by policies or by tariff "cells" in order to build a multifactor net premium model.

The aim of the study is to develop theoretical principles and practical applications in the field of the development and justification of actuarial methods for calculating insurance tariffs while considering future loss reserves.

Methodology. This study examines the independent normalized loss increments and the Bornhuetter-Ferguson models, which resemble an "averaging" of the expected loss amount estimates. It also explores chain-ladder-based models, the Bühlmann-Straub model, and models based on cross-parameterization of normalized loss increment to improve and apply them practically in insurance tariff determination.

Results. The research has shown that actuarial methods remain essential tools in the development and justification of insurance tariffs, as well as in risk mitigation strategies. Based on the findings, a method for estimating late loss reserves using multifactor models that account for the structure of the rated risks has been proposed. The proposed method application allows for a more accurate consideration of individual factors affecting the overall insurance tariff and its net premium component in particular. The results of the analysis based on these models

demonstrated that an attempt to assess the quality of the insurance event period, which is taken into account by the chain ladder, Bühlmann-Straub or cross-parameterization models, can also be carried out by adding the event period factor to the set of factors used.

Practical significance. The study results can be valuable for analysts and underwriters in insurance companies aiming to improve risk assessment and loss reserve planning, especially when the final loss amount is unknown at the time of reserve formation. The obtained findings will help account for factors influencing loss origination in insured events and enable a more precise construction of a unified tariff model in the future. **Prospects for further research** involve expanding the use of mathematical and actuarial methods in tariff justification of an insurance company, considering probable risks under conditions of increased uncertainty in the current domestic environment.

Keywords: insurance, actuarial calculations, loss reserve, late losses, normalized loss increment, generalized linear model, net premium.

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АКТУАЛЬНІ МЕТОДИ ОБЧИСЛЕННЯ СТРАХОВОГО ТАРИФУ У КОНТЕКСТІ РЕЗЕРВУВАННЯ МАЙБУТНІХ ЗБИТКІВ

Анотація

У реаліях сьогодення досить ймовірним є виникнення недоліків існуючих методів оцінки резервів страхових збитків з погляду подальшого застосування багатофакторних моделей для обчислення тарифу. З цією метою виникає необхідність запропонувати підхід до резервування збитків з метою актуарної тарифікації на основі узагальнених лінійних моделей. Також слід наголосити на проблемі розподілу резерву збитків, оціненого актуаріями з використанням відомих моделей резервування, за полісами або за тарифними «осередками» з метою побудови багатофакторної моделі нетто-тарифу. У статті було розглянуто з метою удосконалення і практичного застосування у визначенні страхового тарифу моделі незалежних нормованих збільшень збитків і модель Борнхюттера-Фергюсона, які схожі на «усереднення» оцінок математичних очікувань розміру збитків, моделі на основі ланцюгових методів, модель Бюльмана-Штрауба, моделі на основі перехресної параметризації нормованого збільшення збитків.

Проведене дослідження показало, що актуарні методи залишаються важливим інструментарієм з розробки й обґрунтування страхових тарифів, а також шляхів елімінування ризиків збитків. За результатами проведених досліджень запропоновано метод оцінки резерву пізніх збитків на основі багатофакторних моделей, які враховують структуру ризиків, що тарифікується. Використання запропонованого методу дозволить з більшою ймовірністю враховувати вплив окремих факторів на розмір страхового тарифу загалом і його нетто-частині зокрема. Результати дослідження можуть бути корисними для працівників аналітичних та андеррайтингових підрозділів страхових компаній, які прагнуть удосконалити процес оцінки ризиків і планування обсягів резервування збитків, остаточний розмір яких невідомий на момент, коли необхідно сформувати резерви. Отримані результати дозволять врахувати фактори, які впливають на походження збитків за страховими подіями і точніше побудувати єдину тарифну модель в майбутньому.

Перспективи подальших досліджень полягають в тому, щоб зосередитись на розширенні сфери використання математичних та актуарних методів при обґрунтуванні тарифів страхової компанії з врахуванням ймовірних ризиків в умовах підвищеного рівня невизначеності в сучасних вітчизняних реаліях.

Ключові слова: страхування, актуарні розрахунки, резерв збитків, пізні збитки, нормоване збільшення збитків, узагальнена лінійна модель, нетто-тариф.

Кількість джерел: 12; кількість таблиць: 2; кількість формул: 5.

Problem statement. In modern non-life insurance practice, the fundamental principles of risk theory are widely used to calculate the net premium. According to this theory, the most probable value of the net premium under a risk-neutral approach is the expected value of the insurance loss for a given risk [1, p. 35]. Risk theory assumes the existence of a homogeneous risks set within an insurance portfolio. Thus, a statistical assessment of this mathematical expectation (net premium) becomes possible. However, there are several practical challenges in this theory application. For instance, the presence of insurance contracts with identical terms does not necessarily mean that losses under these contracts can be accounted for as observed values of the same random variable. Insurance contracts typically start on different dates, leading to variations in the probability of insured events occurring at the time of assessment. One contract, for example, may have been in effect for nine months within the actuarial year under review, while another may have been active for only one month.

Another well-known issue is the delayed recognition of losses for events occurring within the analyzed period, which currently has a certain actuarial solution. A common situation is that the final sum of claims payments for a policy year remains unknown even at the end of the current year. This may be due to the fact that the settlement of certain claims can take a significant amount of time (several years) due to lengthy court proceedings or prolonged assessment of insurance compensation amounts. There are also cases where a certain period passes between the occurrence of an insured event and the manifestation of its consequences. All of the above necessitates the use of specific actuarial methods to create reserves for incurred but not reported (IBNR) losses. The purpose of loss reserving is to estimate the reserve for unpaid losses. The losses under consideration are those known to exist but whose final amount is uncertain at the time the reserves need to be established. This paper will discuss the issue under consideration and the existing shortcomings of the current approach.

Analysis of recent research and publications. First, let us review the scholars' opinions on the most popular models for actuarial assessment of late-reported losses and the challenges arising in their practical application under tariff setting using multifactor models. Subsequently, we will attempt to propose solutions to these challenges. According to N. Zinchenko, one of the requirements for modern actuarial practice is ensuring the consistency of actuarial assumptions and their connection with the components of the applied methodology, as well as adherence to the requirements of coherence, completeness, and reliability of input data [3].

Beard R.E., Pentikäinen T. and Pesonen E. believe that these requirements are best met by a unified set of actuarial assumptions (actuarial basis), which is applied universally across all models used to address the full spectrum of actuarial tasks [4]. A concept of a generalized actuarial basis has been proposed, which satisfies these conditions and serves as a unified framework for both general insurance and life insurance contracts. Ryleyev S. V., Bahriy K. L. and Drin I. I. substantiated the relevance of operational economic analysis as a component of management analysis, which allows for obtaining necessary information about the current state of business processes and their outcomes for decision-making on timely interventions and operational management [5, p. 62].

Zubchenko V. and Yamnenko R. argue that the actuarial basis unification has opened opportunities for the broad application of actuarial science and practice in life insurance compared to general insurance, and vice versa. Specifically, a methodology for calculating insurance tariffs [6, p. 83] based on statistical modelling has been developed. This methodology enables the consideration of changes in the insured amount, probabilities (intensities) of an insured event, probabilities (intensities) of an alternative event, and changes in the value of money over time in cases of deferral and installment payments. The proposed methodology can also be applied to contracts that involve risk-sharing – the franchise, partial insurance, insurance, and reinsurance. For typical insurance risks in both general and life insurance, this methodology provides tariff values close to those calculated using standard methods [6].

The study of new sequential and systematic approaches to organizing operational and prospective analysis, incorporating actuarial aspects to enhance the efficiency of analytical algorithms, is the focus of Yu. Manachynska's work [7]. In the meantime, A. Chornovol and Yu. Tabenska have proposed directions for the Ukrainian insurance market further development to ensure its stability and solvency through the formation of insurance portfolios based on well-founded insurance tariffs [8].

The purpose of this article is to study the problem of multifactor models application for calculating insurance tariffs and to develop approaches for loss reserving aimed at actuarial tariff setting based on generalized linear models. This will allow for a more accurate consideration of the individual tariff factors impact on the processes of future loss escalation from past events.

Basic research material presentation. The incurred but not reported (IBNR) claims reserve is considered an estimate of the insurer's liability for insurance payments, including loss adjustment expenses, related to insurance events that occurred in the reporting and previous periods but were not reported to the insurer by the reporting date in accordance with the procedure established by the Ukrainian legislation or the insurance contract. IBNR estimation models have been thoroughly examined in different sources, as well as in methodologies and guidelines for insurance companies [3, 6, 9].

A review of the most popular actuarial methods for calculating incurred but not reported claims reserve enables to highlight the following:

- Modification of the chain-ladder method;
- Model of independent normalized loss development;
- Fixed percentage method;
- Bornhuetter-Ferguson method;
- Cape Cod method;
- Munich chain-ladder method;
- Bühlmann-Straub model;
- Models based on cross-parameterization of normalized loss development;
- Modifications of the aforementioned actuarial methods.

The independent normalized loss development model and the Bornhuetter-Ferguson method resemble an "averaging" of expected loss estimates for late-reported claims over event periods. Models based on chain-ladder methods, the Bühlmann-Straub model, and cross-parameterization approaches define differences between event periods one way or another. These models allow for the derivation of adjustment coefficients for the event period. Moreover, previous research [10] has demonstrated that chain-ladder-based models can be represented as a special case of models based on cross-parameterization of normalized loss developments – specifically, a generalized linear model for cross-parameterization of normalized loss developments following a Poisson distribution [11].

A common drawback of all these approaches in modern conditions can be described as follows. Maintaining competitiveness in the insurance market often requires the construction of multifactor models aimed at more detailed risk tariffing. In practice, when building a multifactor model, the first step is to obtain the following table (Table 1).

Table 1

Dataset for Building a Multifactor Net Premium Model

Factor 1	*Factor 2*	…	*Factor i*	*Risk Exposure*	*Average Losses (Frequency)*
Level 1 of the Factor 1	Level 1 of the Factor 2	…	Level 1 of the Factor *i*	w1	y1
…	…	…	…	…	…
Level *n* of the Factor 1	Level *n* of the Factor 2	…	Level *n* of the Factor *i*	w*i*	y*i*

As a rule, when the number of factors is large, the exposure (Table 1) in most cells w*i* is too small to reliably estimate incurred but not reported (IBNR) losses using the aforementioned models, as the variance of such an estimate would be unacceptably high from a practical standpoint.

A practical solution to this issue is to smooth the impact of individual factors on the amount of late-reported losses by performing the estimation based on the entire exposure volume (or large subsets thereof) and distributing the unknown losses at the tariff calculation date across the cells in Table 1 based on risk exposure.

It is evident that under this approach (using car insurance as an example), the probability of a future loss report for past-year events from a taxi owner is higher than from a motorcycle owner. However, given equal risk exposure, both would receive the same estimate for future reported losses. This could slightly distort the tariff, potentially underpricing taxi insurance while overpricing motorcycle insurance.

Next, let us explore ways to improve the tariff in these circumstances, considering the above factors and using the framework of generalized linear models. Thus, we will assess the reserve for incurred but not reported (IBNR) losses using generalized linear models [12].

The theoretical basis for the proposed approach is a simple IBNR estimation model – the model of independent normalized loss increments. The main reasons for choosing this model are outlined below.

Let us examine the loss development triangle (Table 2).

Table 2

Loss Development Triangle

Event period (i) / development (k)	*1*	*2*	*...*	*i-1*	*i*
1	S_{11}	S_{12}	...	$S_{1.i-1}$	S_{1i}
2	S_{21}	S_{22}	...	$S_{2.\,i-1}$	
...	...	...	...		
i-1	$S_{i-1.1}$	$S_{i-1.2}$			
i	S_{i1}				

The symbol S_k is conventionally used to denote the increase in losses from events in period *i* over the development period *k* (either the amount of claims in the period, or the amount of payments). The period is typically considered as one year.

Within the independent normalized loss increment model, all S_{ik} are assumed to be independent random variables [2, p. 205-206], and the indicators S_{ik}/w_i. have the same expected mathematical value *mk* and variance that differs only due to the risk size (risk exposure), where $ς^2{}_k/w_i$, and $ς^2{}_k$ is a common dispersion parameter for all event periods.

Thus, the proposed factor-based refinement consists of the following: we consider S_{ik} as the sum of losses across factor levels used for tariff calculation (see Table 1):

$$S_{ik} = S_{ik\,1} + \dots + S_{ikn}. \quad (1)$$

where S_{ik1}, S_{ikn} represent the loss increments in the development period *k* corresponding to the intersection of factor levels in the 1st and *n*-th cells of Table 1, respectively.

The expected value of the normalized loss increment in the development period *k*, based on equation (1), is determined by formula (2):

$$E\left(\frac{Sik}{wj}\right) = E\left(\frac{Sik1}{wi1}\right)\frac{wi1}{wi} + \cdots + E\left(\frac{Sikn}{win}\right)\frac{win}{wi}, \qquad (2)$$

where *E* is the mathematical expectation operator.

As seen from formula (2), the mathematically expected value of the normalized loss increment in period *k* varies for different event periods due to the varying structure of $i\ w\ i\ 1/w\ i$ in the portfolio for each event period (which in practice cannot remain the same across all periods).

Next, we assume, as in the independent normalized loss increment model, that the expected value of the normalized loss increment remains constant in a specific cell *j* of Table 1 from period to period, meaning the following is true (3):

$$E\left(\frac{Sikj}{wij}\right) = mkj \qquad (3)$$

As noted earlier in the Table 1 discussion part, due to the small exposure in some cells, it is impossible to accurately estimate the parameters in formula (3) using the independent normalized loss increment model, as proposed by Th. Mack [2, p. 206]. Therefore, we proceed to the next step.

We introduce the assumption that the generalized linear model m_{kj}. depends on factors.

The vector of expected normalized loss increment for individual cells in Table 1 is determined by formula (4):

$$\overrightarrow{m_k} = \begin{pmatrix} m_{k1} \\ \cdots \\ m_{kn} \end{pmatrix} = \begin{pmatrix} g^{-1}(\theta_{k1}) \\ \cdots \\ g^{-1}(\theta_{kn}) \end{pmatrix}, \qquad (4)$$

where g is the canonical link function [4, p. 21] for the distribution of the normalized loss increment under consideration,

θ_{k1}, θ_{kn} are the canonical parameters of the distribution for the normalized loss increment in the *i*th and *n*th cells, respectively.

Thus, the vector of canonical parameters, based on the generalized linear model framework, is defined by formula (5):

$$\overrightarrow{\theta_k} = X_k \beta_k, \quad (5)$$

where X_k is the design matrix for the model in the development year k, representing the assignment of the ith observation (row in Table 1) to the factor level corresponding to the coefficient for that level in the matrix column (with 0 or 1) for categorical factors or numerical values for continuous factors.

β_k is the vector of model coefficients for categorical factor levels and numerical factor coefficients for the model in the development year k.

Regarding the dispersion parameter of the chosen exponential family distribution, the assumption of the independent normalized loss increment model holds: σ_k^2/wij, where σ_k^2/wij is the overall dispersion parameter for all event periods and factor levels. However, in the case of generalized linear models, the variance is now a function of the parameter σ_k^2/wij, and its equality to this parameter holds only for normally distributed values *Sikj/wij*.

The estimation of the generalized linear models parameters can be performed using any available methods, which are now widely accessible to actuaries.

Conclusions of this study and further research prospects. The given estimate of β_k model parameters for all development periods except the first will allow replacing unaccounted late-reported losses with estimates derived from these models. These estimates will better reflect the factors influencing the origin of losses from insurance events and will enable a more accurate construction of a unified multifactor tariff model in the future. An attempt to account for the quality of the event period, as assumed in chain-ladder models, Bühlmann-Straub models, or cross-parameterization, can also be made by adding the event period as a factor in the set of factors used.

As a result of this work, a method for estimating the late-reported losses (IBNR) reserve based on a multifactor model has been proposed. In the long run, this approach will allow for a more precise consideration of the factor structure of the risk being rated. The application of this method will enable actuaries to more accurately account for the influence of individual factors on the net premium rate.

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